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Non-Frictional Jamming of Inclusions—an Ignored Toughening Effect

H.-J. Weiss

Institut für Werkstoffphysik und Schichttechnologie, Helmholtzstr. 20, O-8027 Dresden, Germany

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Abstract

The load carried by inclined platelets bridging a crack is approximately calculated in the absence of bonding and friction in order to separate the effect of non-frictional jamming from that of frictional load transfer. This is done by means of a simple model where the strain field is essentially reduced to uniaxial compression of certain matrix regions. The work of opening the crack against the jamming force provides a contribution to the fracture toughness of the composite. The estimate shows that this contribution may be of practical interest for ceramics.

Die Kraftübertragung schrägliegender Plättchen beim Überbrücken eines Risses wird näherungsweise für den Fall nicht vorhandener Bindung und Reibung berechnet, um den Effekt des nicht-reibungsbedingten Verklemmens von der reibungsbedingten Kraftübertragung abzusondern. Das wird mittels eines einfachen Modells erreicht, bei dem das Deformationsfeld im wesentlichen auf einachsige Kompression gewisser Matrixbereiche reduziert ist. Die gegen das Verklemmen aufzubringende Rißöffnungsarbeit liefert einen Beitrag zur Bruchzähigkeit des Verbundes. Die Abschätzung zeigt, daß dieser Beitrag von praktischem Interesse für Keramik sein könnte.

Le transfert de charge par des plaquettes inclinées pontant une fissure est calculé approximativement, en l'absence de liaison et de frottement, de façon à différencier l'effet du coincement sans frottement dans celui du transfert de charge par frottement. Ceci est réalisé par un modèle simple où le champ de contraintes est réduit essentiellement à une compression uniaxiale pour certaines régions de la matrice. Le travail d'ouverture de la fissure opposé à la force de coincement contribue à un changement de la valeur de la ténacité. Une approximation nous montre que cette contribution peut être d'un intérêt pratique pour les céramiques.

1 Introduction

Fibres or platelets bridging a crack resist its propagation and thus increase the strength of the material. This well-known fact has become a subject of continual research. The wish to obtain analytical and numerical results has produced a variety of composite models with parallel reinforcing elements, although, in reality, the orientation is more or less random in most cases. The results show that with increasing load the interface between matrix and inclusion eventually debonds, whereupon the reinforcing element takes up load by friction, as intended.

Sophisticated pull-out models took into account residual stresses, lateral contraction, changing friction coefficient, and other subtleties. The outcome of those efforts has been ambivalent. On the one hand, useful qualitative and quantitative knowledge has been gathered. On the other hand, reinforcing mechanisms less suitable for numerical approach seem to have been pushed into oblivion. Those which work also in the absence of bonding and friction are denoted here by the term non-frictional jamming in order to set them apart from frictional pull-out. Let us briefly recall them. Mode II and Mode III crack loading are not considered here, so the following is restricted to the less obvious but more important non-frictional effects in Mode I loading.

There is the trivial case represented by fibres with paddle-shaped ends. Then there are the widely applied and less trivial cases of fibres with hooked ends. They would carry load also in the absence of friction, no matter whether they were purely elastic, purely plastic, or made of a real elastic/plastic or other material.

The idea of straight frictionless fibres carrying load is seldom realized by materials scientists. It is found implicitly formulated in literature, in the disguise of energy balances: ductile inclined fibres bridging a crack dissipate energy in plastic deformation when the crack is opened. Consequently, there must be some load carried across the gap. That contribution was taken into account by Helfet & Harris.[1] However, their figure illustrating the effect is misleading as it suggests that one end of the fibre must be fixed. The non-frictional ductile term was also considered in connection with steel fibre concrete,[2] for instance.

Perhaps the most peculiar reinforcement is that by non-ductile straight frictionless fibres or platelets. This mechanism is not so exotic in itself but has rather been made to seem so by decades of frictional pull-out literature. Its absence in conventional pull-out models can be revealed by an easy test: take the result and put the coefficient of friction equal to zero. If the result reduces to zero, non-frictional effects are missing. It is the subject of this paper to draw attention to these effects, and to provide an estimate for the case of platelets.

A theory of frictionless inclusions might be considered futile since in reality there is always some friction. It still makes sense, however, because the jamming effect is more easily and convincingly derived for the frictionless case. Of course the effect is also there in the presence of friction where it acts in addition to the conventional reinforcing mechanisms.

2 Calculating the Effect

Let us first consider the load carried by platelets bridging the crack.[3] There is reason to assume that this is not one of those particular problems whose solution depends strongly on dimensionality. Therefore a two-dimensional approach is chosen with the aim of obtaining the simplest quantitative description of the phenomenon.

Imagine a platelet of length l taken out of the matrix, the matrix cut into halves, and the halves separated by δ (Fig. 1). This gives rise to a mismatch,

$$2u = \delta \cdot \sin\alpha \approx \delta \cdot \alpha \qquad (1)$$

Now the rigid platelet is thought to be jammed into the mismatched half cavities. Thereby, each matrix half gets a local displacement u at the pushed edge,

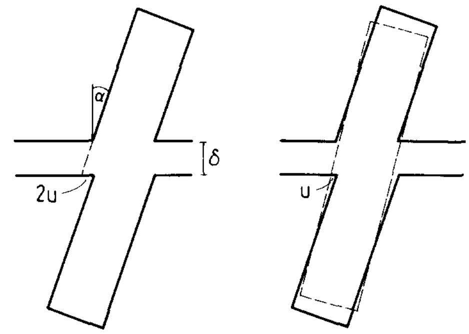

Fig. 1. Visualizing the relation between matrix displacement, u, platelet tilt angle, α, and gap width, δ.

provided that $\delta \ll l$. The displacement is made up of two contributions related to the translational and the rotational part of the matrix displacement field in the vicinity of the platelet,

$$u = u_{tr} + u_{rot} \qquad (2)$$

In order to estimate the forces related to these displacements, an average matrix region is assigned to every platelet in Fig. 2. Its width is easily found as $(1 - v)d/2v$, where v is the volume fraction of platelets.

The model is further simplified in Fig. 3. There is no inconsistency in deriving the displacement from Fig. 1 but the forces from Fig. 3. The shaded matrix areas are thought to be under uniaxial compression, which means that the deviations from uniaxiality near the boundaries of the shaded areas are neglected. From Fig. 3 one can easily derive the force

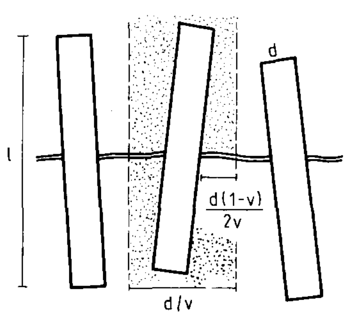

Fig. 2. Assigning an average matrix region to the platelet.

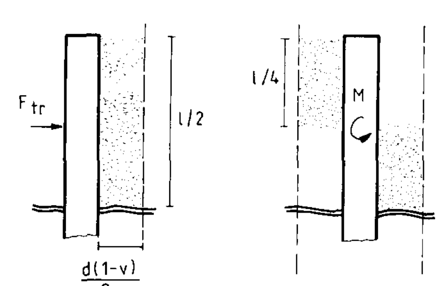

Fig. 3. Compressed matrix regions related to the translational and rotational parts of platelet displacement.

(F_{tr}) and the moment (M) by which the matrix half space and the half platelet act upon each other:

$$F_{tr} = \frac{v}{1-v}\frac{l^2}{d}Eu_{tr} \tag{3}$$

$$M = \frac{1}{12}\frac{v}{1-v}\frac{l^3}{d}Eu_{rot} \tag{4}$$

(The width of the platelet perpendicular to the paper plane has also been taken as l, which is not really a deviation from the two-dimensional approach. E = matrix Young's modulus.)

All the moments acting upon the platelet are in equilibrium:

$$F_{tr} \cdot \frac{l}{4} = M \tag{5}$$

From eqns (2) to (5) follows that u_{tr} is only one fourth of the total displacement,

$$u_{tr} = \frac{u}{4} \qquad u_{rot} = \frac{3u}{4} \tag{6}$$

The normal component of F_{tr} with respect to the crack is the load that one platelet can carry across the crack. With eqns (1), (3) and (6), it can be written as

$$F = F_{tr} \sin\alpha \approx F_{tr} \cdot \alpha = \frac{1}{8}\frac{v}{1-v}E\alpha^2 l^2 \frac{\delta}{d} \tag{7}$$

By assigning an average crack area to every platelet, ld/v according to Fig. 2, from eqn (7) the effective stress carried by the platelet of tilt angle α across the crack is obtained:

$$\sigma = \frac{1}{8}\frac{v^2}{1-v}E\alpha^2\frac{l\delta}{d^2} \tag{8}$$

The finite strength of the components limits the validity of eqn (8) to values δ below a certain δ_m. The work per area consumed in opening the crack from $\delta = 0$ to $\delta = \delta_m$ contributes to the effective fracture toughness of the composite material:

$$\gamma_{(P)} = \int_{\delta=0}^{\delta_m} \sigma \, d\delta = \frac{1}{16}\frac{v^2}{1-v}E\alpha^2\frac{l\delta_m^2}{d^2} \tag{9}$$

(The notation $\gamma_{(P)}$ is to avoid confusion with the work of fracture of the platelets themselves.)

For simplicity only the case of non-breaking platelets and a matrix with the fracture strain $\varepsilon_{Mf} = \sigma_{Mf}/E$ are considered here. The compressional matrix deformation whose relation to δ is derived from eqn (1) and the configuration of Fig. 3,

$$\varepsilon_M = \frac{v}{1-v}\alpha\frac{\delta}{d} \quad \text{for} \quad \frac{v}{1-v} > \frac{2d}{l} \tag{10}$$

must keep below ε_{Mf} lest the matrix be damaged. From this condition follows a maximally admissible δ:

$$\delta_m = \frac{1-v}{v}\cdot\frac{\sigma_{Mf}}{E}\cdot\frac{d}{\alpha} \tag{11}$$

The inequality in eqn (10) represents the condition that the average width of matrix assigned to every platelet, $d(1-v)/2v$, should be smaller than $l/4$. Only then does eqn (10) serve as an approximation for ε_M.

Incidentally, eqn (11) removes the angular dependence from eqn (9).

$$\gamma_{(P)} = \frac{1-v}{16}\cdot\frac{\sigma_{Mf}^2}{E}\cdot l \tag{12}$$

so there is no need for averaging with respect to the angle. Independence of angle means that under the given conditions, differently inclined platelets contribute to fracture toughness in an equal way. Obviously, the validity of this result must be limited since platelets with $\alpha = 0$ certainly do not contribute. Platelets with α near zero are pulled out completely before they can contribute their share to eqn (12). In order to make sure whether eqn (12) can still serve as a useful approximation, the restriction of validity has to be quantified. Only those platelets contribute fully to eqn (12) whose δ_m from eqn (11) is sufficiently smaller than their diameter l. $\delta_m < l/4$ seems to be a reasonable condition for this estimate. Combined with eqn (11) it yields a lower boundary for α in the above sense. An upper boundary for α with a given volume fraction follows from the limited matrix space available for every platelet, as visualized in Fig. 2. Thus eqn (12) refers only to platelets with tilt angles within the bounds

$$4\frac{1-v}{v}\frac{\sigma_{Mf}}{E}\frac{d}{l} \lesssim \alpha \lesssim \frac{1-v}{v}\frac{d}{l} \tag{13}$$

Since the matrix strength/modulus ratio is very small, the lower boundary is also small. Therefore the fraction of platelets with tilt angles below the lower boundary is usually negligibly small, except for very narrow angular distribution. Thus, eqn (12) will be valid for not-too-well aligned platelets, otherwise it has to be modified in a way obvious from the above considerations.

As another restriction, small volume fractions have to be excluded, according to the inequality (10). An effect would be there, of course, also with small volume fraction, but it would not be governed by eqn (12).

The platelets replace the matrix with its fracture toughness $\mathscr{G}_c$, therefore the amount $v\mathscr{G}_c$ has to be subtracted from eqn (12) in order to obtain the net effect of the presence of the platelets.

$$\Delta\mathscr{G}_c = \gamma_{(P)} - v\mathscr{G}_c \tag{14}$$

Expressing the local matrix strength by $\mathscr{G}_c$ and by the size of microcracks, c_0,

$$\sigma_{Mf} = \sqrt{\mathscr{G}_c E/c_0} \tag{15}$$

from eqns (12) and (14)

$$\Delta\mathscr{G}_c/\mathscr{G}_c = \frac{1-v}{16}\frac{l}{c_0} - v \tag{16}$$

is obtained. This may be written as a factor by which the composite fracture toughness is expected to be higher than that of the matrix:

$$\mathscr{G}_{c,\text{composite}} = (1-v)\left(1 + \frac{1}{16}\frac{l}{c_0}\right)\mathscr{G}_c \tag{17}$$

In a similar way, results can be derived for other cases, as platelets of finite strength and stiffness. It is mentioned here without proof that in those cases an effect of similar magnitude has to be expected as derived here for the simple case of rigid non-breaking platelets.

3 Discussion

Obviously the results derived here are meant to be rough estimates. There are several uncertainties involved. One of them is in the value of matrix strength to be inserted into eqn (12). The macroscopic strength is generally too low for this purpose, since it is governed by the largest flaws. An appropriate value would be the matrix strength within the small regions between the platelets. This local strength is usually not known but may be estimated from measured quantities.

The matrix between the platelets is subjected to uniaxial compression in this approach. The simplification is justified when estimating the force carried by the platelet. It would not be justified, however, to take a local compression strength for σ_{Mf} in eqn (12) because the stress is not uniaxial in the vicinity of the platelet, with tensile components being always present. That tensile stress may cause damage starting from microcracks of size c_0. By choosing the representations (16) or (17) instead of eqn (12), the uncertainty of σ_{Mf} is replaced by the uncertainty of c_0. This is not a peculiarity of the present model but the usual uncertainty inherent in brittle fracture mechanics.

With due reservations concerning the quantitative aspect of the results, the following can be stated:

Platelets or straight fibres are able to carry load also in the absence of bonding and friction. This non-frictional jamming has apparently been largely ignored. In the case of platelets, and within certain bounds, it depends weakly on volume fraction and is independent of orientation and thickness, to a first approximation.

There is another problem which tends to be neglected in discussing fracture toughness. The elastic energy released by breaking inclusions or matrix microcracks is often regarded as dissipated energy contributing to fracture toughness. This is not necessarily so. The surplus energy released in unstable cracking of a component moves about the sample as elastic waves, which may initiate more local cracking and further the propagation of the main crack. This undesirable mechanism has not been considered here. It is believed to somewhat but not essentially reduce the toughening effect described here.

Finally the result derived in this paper is illustrated by the following example. For reinforcement of ceramics, platelets with $l/d \approx 10$ seem to be suitable for practical reasons.[4] With this number, the inequality (10) required $v > 0{\cdot}17$ as a condition for the validity of the present model. With an assumed local matrix strength $\sigma_{Mf} = E/200$ and $v = 0{\cdot}25$, eqn (13) provides the information that platelets with tilt angles less than 0·35° do not fully contribute, and that there is room for angles up to 17°. So the number of platelets with $\alpha < 0{\cdot}35°$ and their reduced contribution may be neglected (except for the particular case of neatly aligned platelets, which would require a modification of eqn (12)). With an assumed modulus of the matrix of 400 GPa and with 100 μm platelets, the contribution of the platelets to the work of fracture follows from eqn (12) as $\gamma_{(P)} = 0{\cdot}5$ N/cm. This is in the order of magnitude of the fracture toughness of sintered alumina, for instance, so that there is a positive net effect in that case, according to eqn (14).

The numerical result is to show that with reasonable assumptions about the components the effect comes out neither unreasonably high nor negligibly small, which is a necessary condition for the usefulness of this approach.

Of course, the result can be of practical relevance only if the platelets are incorporated into the matrix without creating large additional defects.

Acknowledgement

N. Claussen proposed the investigation of the reinforcing effect of poorly bonded platelets.

References

1. Helfet, J. L. & Harris, B., Fracture toughness of composites reinforced with discontinuous fibres. *J. Mater. Sci.*, **7** (1972) 494–8.
2. Brandt, A. M., On the optimization of the fibre orientation in cement based composite materials. In *Present State of Investigations and Applications of Fibre-reinforced Cement-based Materials*, ed. Z. Jamrozy. Cracow, 1983, pp. 13–22.
3. Weiss, H.-J., An estimate of the reinforcing effect of debonded frictionless platelets. In *Brittle Matrix Composites 3 (Proc. 3rd Int. Symposium, Warsaw 1991)*, ed. A. M. Brandt & I. M. Marshall. Elsevier Applied Science, London, 1991, pp. 328–34.
4. Claussen, N., Ceramic platelet composites. In *Structural Ceramics—Processing, Microstructure and Properties (Proc. 11th Riso International Symposium on Metallurgy and Materials Science)*, ed. J. J. Bentzen, J. B. Bilde-Sorensen, N. Christiansen, A. Horsewell & B. Ralph. Riso National Labs, Roskilde, Denmark, 1990, pp. 1–12.